CALCULATION OF THE NONSTATIONARY TEMPERATURE STRESSES IN A PLATE OF VISCOELASTIC MATERIAL WHOSE PROPERTIES ARE NOT SUBJECT TO THE PRINCIPLE OF THE TIME-TEMPERATURE ANALOGY

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Equations are constructed for determining the relative strains and stresses in an infinite plate of viscoelastic material exposed to symmetrical heating. Possible methods of solving these equations are examined in relation to various models.

The application of the time-temperature superposition principle considerably simplifies the solution of problems of the theory of thermoviscoelasticity. However, for certain structural plastics, including a number of glass-reinforced plastics, this principle is either inapplicable or can be applied only within a very limited temperature range. For these materials it is necessary to obtain a solution with allowance for the temperature dependence of the mechanical characteristics of viscosity and elasticity.

We will consider one of the simplest of these problems. An infinitely long plate (Fig. 1) of width 2R is subjected to nonstationary heating symmetrical about the X-axis. In this case the cross sections of the plate are free of bending moments about the Y-axis, and the relative strains in the direction of the longitudinal X-axis do not depend on the Z coordinate and are equal (ε_x = const).

We will determine the time dependence of the relative strains and stresses in the plate in the directions of the axes X, Y, Z.

1. Hooke material. For this material the stress-strain relation for uniaxial states of stress has the form

$$\sigma = E\varepsilon - E\alpha T. \quad (1)$$

Applying this relation to our problem, we obtain

$$\sigma_x = E\varepsilon_x - E\alpha T, \quad \sigma_y = E\varepsilon_y - E\alpha T = 0,$$

$$\sigma_z = E\varepsilon_z - E\alpha T = 0. \quad (2)$$

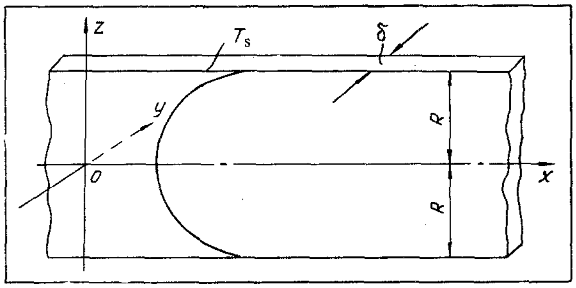

Fig. 1. Model of temperature action on plate.

From the first of Eqs. (2) we find that in the absence of external forces $P_x = P_y = P_z = 0$ with allowance for heating symmetry

$$\int_0^R E\varepsilon_x dZ - \int_0^R E\alpha T dZ = P_x = 0. \quad (3)$$

We will denote mean integral values of the quantities over the width of the plate by a bar; then from Eq. (3) with α = const we find that

$$\varepsilon_x = \frac{\alpha\int_0^R ETdZ}{\int_0^R EdZ} = \frac{\alpha\overline{ET}}{\overline{E}}. \quad (4)$$

Similarly, for the other two directions $\varepsilon_y = \alpha\overline{T}$; $\varepsilon_z = \alpha\overline{T}$.

In these expressions the temperature T is a function of the coordinate X and time t: T = T(Z,t). In accordance with (2),

$$\sigma_x = E\alpha\left(\frac{\overline{ET}}{\overline{E}} - T\right),$$

$$\sigma_y = 0, \quad \sigma_z = 0. \quad (5)$$

In the particular case with E = const it follows from (5) that

$$\sigma_x = E\alpha(\overline{T} - T)^{*)}. \quad (6)$$

Using (6) we can obtain the analytic dependence $\sigma_z(Z,t)$, if the law of temperature vaiation T(Z,t) is given. Thus, for boundary conditions of the first kind the exact solution of the problem has the form [1]

$$\sigma_x(Z,t) = E\alpha(T_0 - T_s)\times$$

$$\times\sum_1^\infty\left(B_n - A_n\cos\mu_n\frac{Z}{R}\right)\exp(-\mu_n^2\,\mathrm{Fo}). \quad (7)$$

Starting from a certain instant of time ($\mathrm{Fo} = \mathrm{Fo}_1 > 0.1$) with a high degree of accuracy we can confine ourselves to a single term of series (7):

*The formulas obtained relate to a maximally thin plate ($\delta \ll R$). If the thickness of the plate is commensurable with the width ($\delta \approx R$), then $\sigma_z = 0$, $\sigma_x = \sigma_y = (E\alpha/(1-\mu))(\overline{T} - T)$.

$$\sigma_x(Z,t) = E\alpha(T_0 - T_s)\times$$

$$\times\left(B_1 - A_1\cos\mu_1\frac{Z}{R}\right)\exp(-\mu_n^2\,\mathrm{Fo}). \qquad (8)$$

2. Maxwell material. The stress-strain relation for this material in uniaxial states of stress is given by

$$E(\dot{\varepsilon} - \alpha\dot{T}) = n\sigma + \dot{\sigma}. \qquad (9)$$

Going over to finite differences, on the basis of (9) for the instant of time t + 1 we can write

$$\sigma_{t+1} = E\varepsilon_{t+1} - E\varepsilon_t +$$

$$+ E\alpha T_t - E\alpha T_{t+1} - n\sigma_t\Delta t + \sigma_t. \qquad (10)$$

We use Eq. (10) to calculate the stresses in the plate. Since there are no external forces,

$$\sum_1^k \sigma_{i,t+1}\Delta Z = P_x = 0 \quad (i = 1, 2, \ldots, k). \qquad (11)$$

On the basis of (10)

$$\varepsilon_{t+1} = \varepsilon_t + \frac{\Delta t\sum_1^k n_i\sigma_{i,t}\Delta Z}{\sum_1^k E_i\Delta Z} +$$

$$+ \frac{\sum_1^k E_i\alpha T_{i,t+1}\Delta Z - \sum_1^k E_i\alpha T_{i,t}\Delta Z}{\sum_1^k E_i\Delta Z}, \qquad (12)$$

whence, again going over to infinitesimals,

$$\dot{\varepsilon}_x = \frac{\int_0^R n\sigma_x dZ}{\int_0^R EdZ} + \alpha\frac{\frac{d}{dt}\int_0^R ETdZ}{\int_0^R EdZ}. \qquad (13)$$

Using our mean-integral notation, we finally write Eq. (13) in the form

$$\bar{E}\dot{\varepsilon} - \alpha\overline{\dot{TE}} = \overline{n\sigma}. \qquad (14)$$

In structure this equation is analogous to relation (9), but contains the averaged values of the corresponding quantities.

Eliminating the relative strain ε_x from (9) and (4), we establish that

$$\frac{\bar{E}}{E}(n\sigma + \dot{\sigma}) + \bar{E}\alpha\dot{T} - \alpha\overline{\dot{ET}} - \overline{n\sigma} - \dot{\bar{\sigma}} = 0. \quad (15)$$

At $\alpha = 0$

$$\frac{\bar{E}}{E}(n\sigma + \dot{\sigma}) - \overline{n\sigma} - \dot{\bar{\sigma}} = 0. \qquad (16)$$

Equation (15) is an integral equation in the unknown function $\sigma(Z,t)$; in the general case it admits of only an approximate analytic or numerical solution. One of the possible approximate methods of solving this equation is the collocation method. In accordance with the collocation procedure [2], the solution of Eq. (15) is found in the form of a sum of known (coordinate) functions:

$$\sigma_m(Z,t) = \sum_1^m C_i\varphi_i(Z,t), \qquad (17)$$

where $C_1, C_2, C_3, \ldots, C_m$ are unspecified coefficients.

Substituting (17) into Eq. (18), we obtain the error

$$R(\sigma_m) = \frac{\bar{E}}{E}[n\sigma_m(Z,t) + \dot{\sigma}_m(Z,t)] -$$

$$- n\bar{\sigma}_m(Z,t) - \dot{\bar{\sigma}}_m(Z,t) + \bar{E}\alpha\dot{T} - \alpha\overline{\dot{ET}}. \qquad (18)$$

We then select the parameters $C_1, C_2, \ldots, C_m$ so that the error $R(\sigma_m)$ vanishes in the given system of points Z_i ($i = 1, 2, 3, \ldots, m$) on the interval $(0, R(Z))$ (collocation points), i.e., we assume that

$$R[\sigma_m(Z_i)] = 0 \quad (i = 1, 2, \ldots, m),$$

$$0 \leqslant Z_1 < Z_2 < \ldots < Z_{m-1} < Z_m < R(Z). \qquad (19)$$

From this condition we obtain an algebraic linear system of equations in the unknowns $C_1, C_2, \ldots, C_m$:

$$R\left[\sum_1^m C_i\varphi_i(Z,t)\right] = 0. \qquad (20)$$

By this method, provided that σ_m converges to the exact solution (as m tends to infinity), we can find the approximate solution with any degree of accuracy by taking a sufficiently large number of parameters $C_1, C_2, \ldots, C_m$.

As an example we will obtain the solution of Eq. (15) for the heating of a plate at constant surface temperature (boundary conditions of the first kind). For simplicity we set E = const and assume that $C_2 = C_3 = \ldots = C_m = 0$. We find the solution (for Fo > 0.1) taking (10) into account in the form in which the elastic solution was obtained:

$$\sigma = E\alpha(T_0 - T_s)\times$$

$$\times C_1\left(B_1 - A_1\cos\mu_1\frac{Z}{R}\right)\exp(-\mu_1^2\,\mathrm{Fo}). \qquad (21)$$

Here, C_1 is an unspecified coefficient.

We substitute (21) into Eq. (17). Making the substitutions

$$B_1 = \frac{8}{\pi^2};\quad A_1 = \frac{4}{\pi};\quad \mu_1 = \frac{\pi}{2},$$

for the collocation point Z = R we find that

$$C_1 = \frac{\pi^2 a}{\pi^2 a + 4R^2\left(\bar{n} - n - \frac{\pi}{2}\,\overline{n\cos\frac{\pi Z}{2R}}\right)}. \quad (22)$$

The value of C_1 is finally calculated from the given temperature dependence of the parameter n of the Maxwell material.

The graphs in Fig. 2 represent the variation of the stresses with time in a plate 10 mm wide at $T_0 = 20°$ C and $T_s = 60°$ C. The mechanical and thermophysical properties of the plate material correspond to the

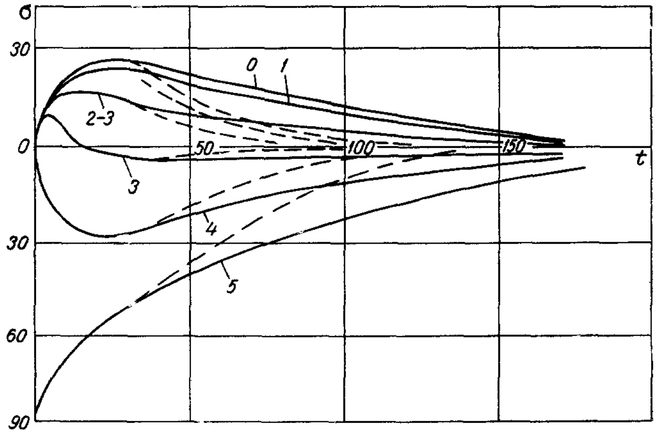

Fig. 2. Variation of the stresses (kg/cm²) with time (sec). The figures on the curves are the coordinates of the points in mm over the thickness of the plate. Solid lines—elastic solution, dashed lines—solution for a Maxwell material.

properties of polymethyl methacrylate. The solid lines represent the stresses calculated from Eq. (8) on the assumption that the material is elastic. The dashed lines correspond to solution (21). As follows from the graph, starting from a certain moment of time (Fo = = 0.1) the stresses in the plate vary exponentially. Taking the viscous properties into account reduces the stress levels at all points of the plate.

3. **Voigt material.** The stress-strain relation for a Voigt material in uniaxial states of stress is

$$\sigma = E(\varepsilon - \alpha T) - \eta(\dot{\varepsilon} - \alpha \dot{T}). \tag{23}$$

To solve the problem we write this equation in finite differences:

$$\sigma_t = E\,\varepsilon_t + \eta\left(\frac{\varepsilon_{t+1} - \varepsilon_t}{\Delta t} - \alpha\,\frac{T_{t+1} - T_t}{\Delta t}\right) - E\alpha T_t\,. \tag{24}$$

Since there are no external forces, by analogy with (13) we write

$$\sum_1^k \sigma_i \Delta Z = P = 0.$$

Summing and going back again from ΔZ to infinitesimals dZ, we find that for symmetrical heating

$$\bar{\sigma} = \bar{E}\varepsilon - \alpha\overline{TE} + \bar{\eta}\dot{\varepsilon} - \alpha\overline{\eta\dot{T}}. \tag{25}$$

In this expression $\bar{\sigma}$ is the mean integral value of the stress in the direction of the X-axis, equal to the external force P divided by the cross-sectional area. If P ≠ 0, the structure of Eq. (25) coincides with that of (23); however, (25) contains the quantities $\bar{E}$ and $\bar{\eta}$ averaged over the width of the plate.

Equation (25) has an exact solution, and namely,

$$\varepsilon = \left[\int_0^t Q\left(\exp\int_0^t \frac{\bar{E}}{\bar{\eta}}\,dt\right)dt + \varepsilon_0\right]\exp\left(-\int_0^t \frac{\bar{E}}{\bar{\eta}}\,dt\right),$$

$$Q = \frac{\alpha\,\overline{TE}}{\bar{\eta}} + \frac{\alpha\,\overline{\eta\dot{T}}}{\bar{\eta}}. \tag{26}$$

At large relative strains measured in tens and hundreds of percent, such as are typical, for example, of thermoplastic molding processes, in view of the smallness of the temperature strains we can assume that Q ≈ 0. Then (26) takes the form

$$\varepsilon = \varepsilon_0 \exp\left(-\int_0^t \frac{\bar{E}}{\bar{\eta}}\,dt\right). \tag{26'}$$

4. **Hereditary material.** The stress-strain relation for a linear hereditary material is established by the expressions

$$\sigma = E(\varepsilon - \alpha T) - E\int_0^t R(t-s)(\varepsilon - \alpha T)\,ds, \tag{27}$$

$$\varepsilon = \frac{\sigma}{E} + \frac{1}{E}\int_0^t K(t-s)\,\sigma\,ds + \alpha T. \tag{28}$$

Each of these equations can be used for describing the σ(t) relation in an individual layer of the plate. We divide the time interval 0–t into t segments Δt. In accordance with (28), for the first time segment 0 + 1 we can write that in the i-th layer

$$\varepsilon_{0+1} = \frac{\sigma_{i,0+1}}{E_i} + \frac{1}{E_i}K_i(\Delta t)\,\sigma_{i,0+1}\Delta t + \alpha T_i. \tag{29}$$

Hence the stress in that layer

$$\sigma_{i,0+1} = \frac{E_i(\varepsilon_{0+1} - \alpha T_i)}{1 + K_i(\Delta t)\Delta t}. \tag{30}$$

Summing the stresses over all the layers, we find that

$$\sum_1^k \sigma_{i,0+1} = \sum_1^k \frac{E_i(\varepsilon_{0+1} - \alpha T_i)}{1 + K_i(\Delta t)\Delta t}. \tag{31}$$

Hence, considering that at t = 1 · Δt, $\lim_{\Delta t \to 0} K_i(\Delta t) \to$ idem,

$$\varepsilon_{0+1} = \frac{P + \sum_1^k \frac{E_i \alpha T_i}{1 + K_i(\Delta t)\Delta t}}{\sum_1^k \frac{E_i}{1 + K_i(\Delta t)\Delta t}}. \tag{32}$$

For the time 0 + 2Δt we correspondingly obtain

$$\varepsilon_{0+2} =$$

$$= \frac{P + \sum_1^k \frac{E_i \alpha T_i}{1 + K_i(\Delta t)\Delta t} + \sum_1^k \frac{K_i(2\Delta t)\sigma_{i,0+1}}{1 + K(\Delta t)\Delta t}}{\sum_1^k \frac{E_i}{1 + K_i(\Delta t)\Delta t}}. \quad (33)$$

We similarly establish that at time 0 + t

$$\varepsilon_{0+t} = \left(\sum_1^k \alpha E_i T_i + P[1 + K_i(\Delta t)\Delta t] + \right.$$

$$\left. + \Delta t \sum_1^k K_i(t\Delta t)\sigma_{0+1}\right)\left(\sum_1^k E_i\right)^{-1} + \ldots$$

$$\ldots + \frac{\Delta t \sum_1^k (2\Delta t)\sigma_{0+(t-1)}}{\sum_1^k E_i}. \quad (34)$$

Increasing the number of intervals ($m \to \infty$ and $\Delta t \to 0$), in the limit we obtain

$$\varepsilon = \frac{P}{\overline{E}} + \frac{1}{\overline{E}}\int_0^t \int_0^R K(t-s)\sigma\, dZ ds + \alpha \frac{\overline{ET}}{\overline{E}}$$

or

$$\varepsilon = \frac{P}{\overline{E}} + \frac{1}{\overline{E}}\int_0^R \int_0^t \overline{K(t-s)\sigma}\, dZ ds + \alpha \frac{\overline{ET}}{\overline{E}}. \quad (35)$$

In structure the latter equation corresponds to the starting relation (28), but contains characteristics averaged over the width of the plate. The analytic solution of this equation for specific temperature conditions can be obtained by the collocation method in the form of a sum of coordinate functions:

$$\sigma_{mn}(Z, t) = \sum_1^m \sum_1^n C_{ij}\varphi_i(t)\psi_j(Z),$$

$$i = 1, 2, 3, \ldots, m;\ j = 1, 2, 3, \ldots, n. \quad (36)$$

As distinct from the solution for a Maxwell material the collocation points are assigned along two axes: on the interval 0–R along the Z coordinate axis and on the interval 0–t along the time axis. Summation with respect to two indices considerably increases the labor of calculation when Eq. (36) is employed. Accordingly, the most effective methods of solving Eq. (35) may be numerical, for example, the method of finite sums.

In calculations based on equations (22), (26), (26'), (36) finding the means $\overline{n}$, $\overline{E}$, $\overline{\eta}$ etc. presents certain difficulties. The procedure can be simplified by approximating the experimental relations n(T), E(T), η(T) with linear functions: $n(T) = n_0 + aT$; $E(T) = E_0 + bT$; $\eta(T) = \eta_0 + cT$.

Averaging over the interval 0–R, we obtain

$$\overline{n} = n_0 + a\overline{T},\ \overline{E} = E_0 + b\overline{T},\ \overline{\eta} = \eta_0 = c\overline{T}.$$

The function $\overline{T}$ has been tabulated for a series of solutions of the heat conduction equation in [1].

In conclusion, we present an example of the complete calculation of the strains and stresses in a polymethyl methacrylate plate on the assumption that the mechanical properties of the material satisfy Voigt's equation.

Conditions of the problem. The plate is initially stretched in the high-elastic state at a temperature T = 130° C to a strain ε_0 = 15.2% and then slowly cooled in the stretched state. These and greater strains occur when polymethyl methacrylate is vacuum-formed. The initial temperature of the plate T = = 20° C is constant over the thickness 2R = 4 mm. At time t = 0 the temperature of the plate surfaces becomes equal to T_s = 100° C and remains constant in time. It is required to calculate the change in the relative deformation of the plate as a function of time. The thermal diffusivity of polymethyl methacrylate $a = 4 \cdot 10^{-4}$ m²/hr.

As the temperature of the plate rises, owing to the action of the internal "frozen" stresses the heating and softening of the material is accompanied by a process of shrinkage and shortening of the dimensions to the initial values. The given temperature interval is quite broad (130–20–100°) and includes the glass transition temperature of polymethyl methacrylate. In the presence of such a considerable temperature change the principle of the time-temperature analogy does not apply. Since the strain component due to thermal expansion is much less than the initial value of the total strain ($\alpha\Delta T = 8 \cdot 10^{-5}$ (100–20°) = 0.64%), Eq. (26') will be used for calculation purposes. We first calculate $\overline{E}$ and $\overline{\eta}$ for various values of the time t. The results of the calculation are presented in the table. We replace the interval in (26') by the sum

$$\int_0^t \frac{\overline{E}}{\overline{\eta}} dt \simeq \sum_0^{t_m} \frac{\overline{E}_t}{\overline{\eta}_t}\Delta t.$$

The calculated values of the integral and the function $\varepsilon(t)$ are also presented in the table. The $\varepsilon(t)$ relation is plotted in Fig. 3, which also gives the results of an experimental verification. A plate of polymethyl methacrylate measuring 17 × 24 × 4 mm was lowered into boiling water, the long dimension of the plate was measured periodically, and the relative deformation calculated. As a comparison shows, best agreement with the calculation is observed at small heating times and also after the plate has been fully heated (Fo > 1). The greatest discrepancy (up to 27%) between the calculated and experimental values of the specimen length is observed on the interval 140–300 sec, which is primarily attributable to the nature of the assumptions made,* and also to the relative inaccuracy of the input data. In spite of this, the relation obtained gives a qualitatively correct reflection of the shrinkage process.

*The linear approximation of E(T) and η(T), the use of the Voigt equation, and the condition a = const.

Variation of the Material Parameters and the Length of the Specimen with Time

No.	$\pi^2 \frac{Fo}{4}$	t, sec	$\bar{\theta}$	$\bar{T}=\bar{\theta}(T_0-T_s)+T_s$, °C	$\bar{E}\cdot 10^{-4}$, kg/cm²	$\bar{\eta}\cdot 10^{-7}$, kg/cm²·sec	$\exp\Sigma \frac{\bar{E}}{\bar{\eta}}\Delta t$	$\varepsilon = \varepsilon_0 \exp\Sigma \frac{\bar{E}}{\bar{\eta}}\Delta t$	Δl, mm	$l=l_0+\Delta l$, mm	$\frac{Fo}{4}$	θ_y	$T_y=\theta(T_0-T_s)+T_s$, °C	$E\cdot 10^{-4}$, kg/cm²	$\eta\cdot 10^{-7}$, kg/cm²·sec	σ_n, kg/cm²
1	0.00	0.00	1	20	3.04	1.52	1	0.152	23	174.	0.000	1.000	20	3.04	1.52	—14.8
2	0.10	11.05	0.773	38.2	2.35	1.18	0.98	0.149	22.5	173.5	0.0101	0.999	20	3.04	1.52	—15.1
3	0.20	23	0.679	45.7	2.07	1.04	0.95	0.145	21.9	172.9	0.0203	0.974	22	2.97	1.48	—15.5
4	0.30	34.5	0.607	51.4	1.85	0.93	0.93	0.142	21.4	172.4	0.0304	0.915	26.8	2.79	1.39	—15.8
5	0.40	46	0.546	56.3	1.66	0.83	0.91	0.138	20.8	171.8	0.0507	0.842	32.5	2.57	1.29	—16.5
6	0.50	57.5	0.493	60.6	1.5	0.75	0.89	0.135	20.4	171.4	0.0406	0.768	38.5	2.34	1.17	—16.2
7	0.60	69	0.145	64.4	1.36	0.68	0.87	0.132	20.0	171	0.0608	0.697	44.3	2.12	1.06	—16.8
8	0.70	81.5	0.403	67.7	1.23	0.62	0.85	0.129	19.5	170.5	0.0709	0.632	49.4	1.93	0.97	—17.1
9	0.80	92	0.364	70.9	1.11	0.56	0.83	0.126	19.0	170	0.0811	0.571	54.3	1.74	0.87	—17.4
10	0.90	103.5	0.330	73.6	1.01	0.51	0.81	0.123	18.6	169.6	0.0912	0.517	58.7	1.57	0.79	—16.7
11	1.00	115	0.298	76.2	0.97	0.47	0.79	0.120	18.1	169.1	0.101	0.469	62.5	1.43	0.72	—16.0
12	1.20	138	0.244	80.5	0.75	0.38	0.76	0.115	17.4	168.4	0.122	0.382	69.4	1.17	0.59	—13.5
13	1.40	161	0.200	84	0.62	0.31	0.72	0.110	16.6	167.6	0.142	0.313	74.9	0.96	0.48	—11
14	1.60	184	0.184	85.3	0.56	0.29	0.69	0.105	15.9	166.9	0.162	0.257	79.4	0.80	0.40	—10.5
15	1.80	207	0.134	89.3	0.41	0.21	0.66	0.100	15.1	166.1	0.182	0.211	83.1	0.65	0.33	—10.0
16	2.00	230	0.110	91.2	0.35	0.18	0.63	0.096	14.5	165.5	0.203	0.171	86.3	0.53	0.27	— 8.4
17	2.20	253	0.090	92.8	0.28	0.13	0.6	0.091	13.7	164.7	0.223	0.140	88.6	0.44	0.22	— 7.9
18	2.40	276	0.073	94.2	0.23	0.12	0.57	0.087	13.2	164.2	0.243	0.115	90.8	5.36	0.18	— 7.3
19	2.60	299	0.060	95.2	0.19	0.1	0.55	0.084	12.7	162.7	0.264	0.094	92.5	0.30	0.15	— 6.6
20	2.80	322	0.049	96.1	0.15	0.09	0.52	0.079	12.0	162.0	0.284	0.077	93.9	0.24	0.13	— 5.8
21	3.00	345	0.040	96.8	0.13	0.07	0.50	0.076	11.5	161.5	0.304	0.062	95.0	0.20	0.10	— 5.6
22	3.20	368	0.033	97.4	0.11	0.06	0.48	0.073	11.0	161.0	0.324	0.052	95.8	0.18	0.09	— 5.4
23	3.50	391	0.024	98.1	0.08	0.05	0.46	0.070	10.6	160.6	0.355	0.038	97.0	0.12	0.07	— 5.0

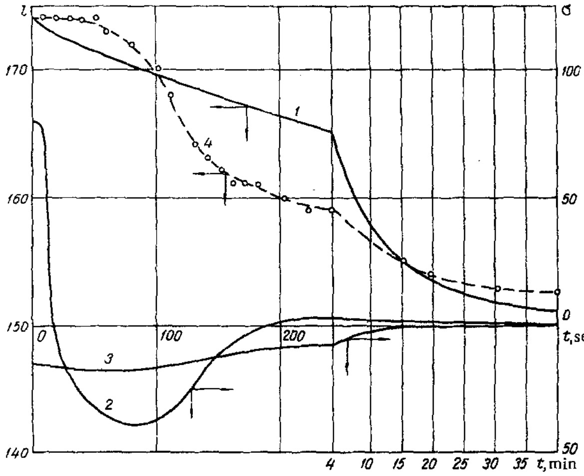

Fig. 3. Variation of the length of a polymethyl methacrylate specimen (mm) and the stresses σ_y and σ_n (kg/cm²) as a result of temperature action: 1) $l = l(t)$; 2) $\sigma_y = \sigma_y(t)$; 3) $\sigma_n = \sigma_n(t)$; 4) experimental results for $l = l(t)$.

In the calculation we employed a linear approximation of the temperature dependence of the mechanical properties of polymethyl methacrylate, namely,

$$E = 3.8\cdot 10^4 - 3.79\cdot 10^2\ T\ \text{kg/cm}^2,$$

$$\eta = 1.9\cdot 10^7 - 1.89\cdot 10^5\ T\ \text{kg/cm}^2\cdot\text{sec}.$$

Obviously, more accurate results can be obtained if, for example, the approximation takes the form

$$E = a_1 + a_2 T + a_3 T^2,$$

$$\eta = b_1 + b_2 T + b_3 T^2.$$

The table also includes the values of the parameters needed for calculating the stresses in the middle surface (σ_y) and the surface stresses (σ_n). The sign of the stress is determined according to the rule

$$\varepsilon \lesseqgtr 0, \quad \sigma(\varepsilon) = E\varepsilon \lesseqgtr 0,$$

$$\dot{\varepsilon} \lesseqgtr 0, \quad \sigma(\dot{\varepsilon}) = \eta\dot{\varepsilon} \lesseqgtr 0.$$

In accordance with (23)

$$\sigma = \sigma(\varepsilon) + \sigma(\dot{\varepsilon}).$$

The time dependence of the stresses σ_y and σ_n is shown graphically in Fig. 3.

The equations obtained above take into account the variation of the mechanical properties of the viscoelastic material with respect to time and the space coordinate. Therefore they can be used for calculating the stresses and strains due not only to temperature effects but also to other factors leading to similar changes, including moisture content [3], aggressive media, radiation, etc., in other words, factors that create a nonstationary field of variation of the mechanical characteristics in a plate of viscoelastic material.

NOTATION

T_0 is the initial temperature of plate; T_s is the surface temperature of plate; μ_n are the roots of characteristic equations; A_n and B_n are constant coefficients; $Fo = at/R^2$ is the Fourier number; k is the number of divisions ΔZ with respect to the Z coordinate; ε_0 is the initial deformation; m is the number of intervals Δt.

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